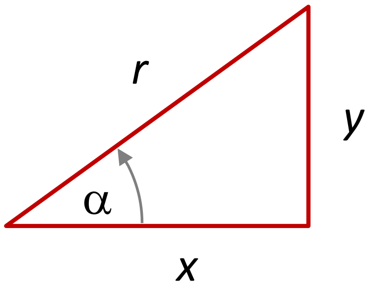
Trigonometrische Relationen

$$\sin\alpha = \frac{y}{r} \qquad \cos\alpha = \frac{x}{r} \qquad \tan\alpha = \frac{y}{x} = \frac{\sin\alpha}{\cos\alpha}$$



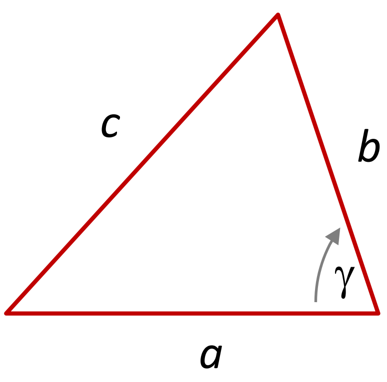
$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha \qquad \sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

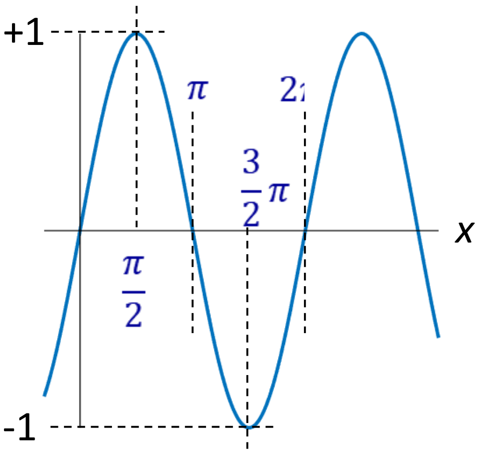
Kosinussatz

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

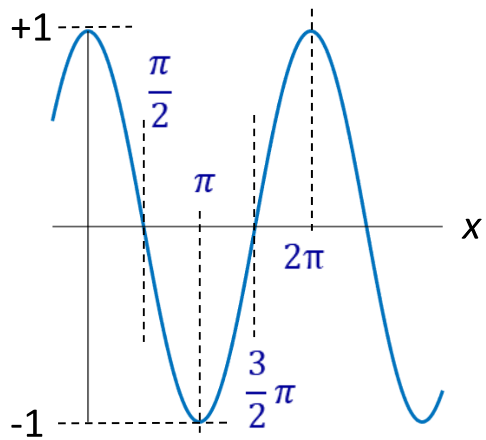


Elementare Funktionen

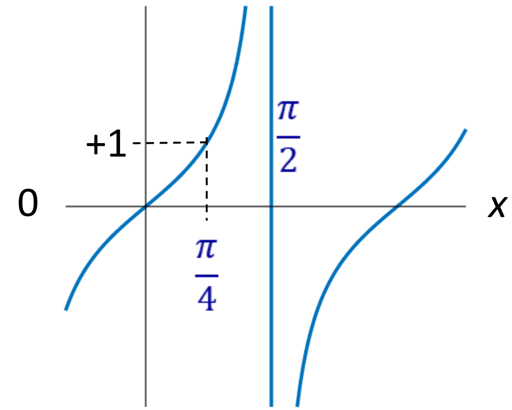
y=sin(*x*)



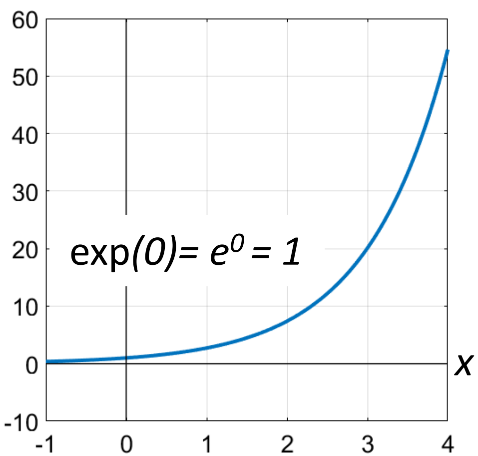
y=cos(*x*)



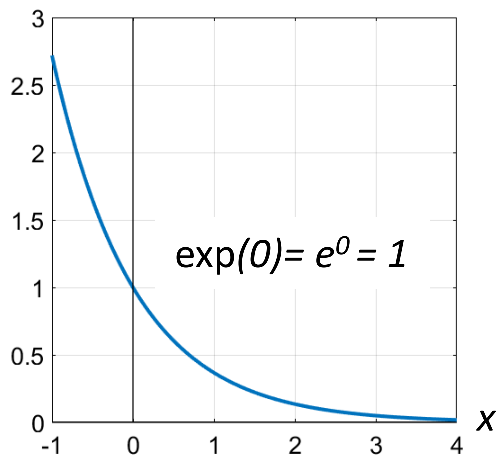
y=tan(*x*)



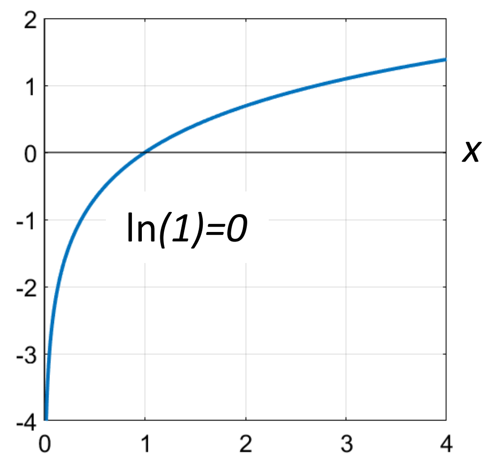
y=exp(*x*)=e^x



y=exp(-*x*)=e^{-x}



y=ln(*x*)



Ableitungen und Integrale

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{1}{n+1}\, x^{n+1}$
$\frac{1}{x}$	$-\frac{1}{x^2}$	$\ln(x)$
$\sin(x)$	$\cos(x)$	$-\cos(x)$
$\cos(x)$	$-\sin(x)$	$\sin(x)$
$\tan(x)$	$\frac{1}{\cos^2 x}$	$-\ln(\cos(x))$
$\exp(x) = e^x$	$\exp(x) = e^x$	$\exp(x) = e^x$
$\ln(x)$	$\frac{1}{x}$	$x\ln(x) - x$

Näherungen/Taylorreihen

$$(1+x)^n \approx 1 + nx + \frac{n(n-1)}{2}x^2$$

$$\frac{1}{1+x} \approx 1 - x$$

$$\sin(x) \approx x - \frac{1}{3}x^3$$

$$\cos(x) \approx 1 - \frac{1}{2}x^2$$

$$\tan(x) \approx x + \frac{1}{3}x^3$$

$$e^x \approx 1 + x + \frac{1}{2}x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

Komplexe Zahlen

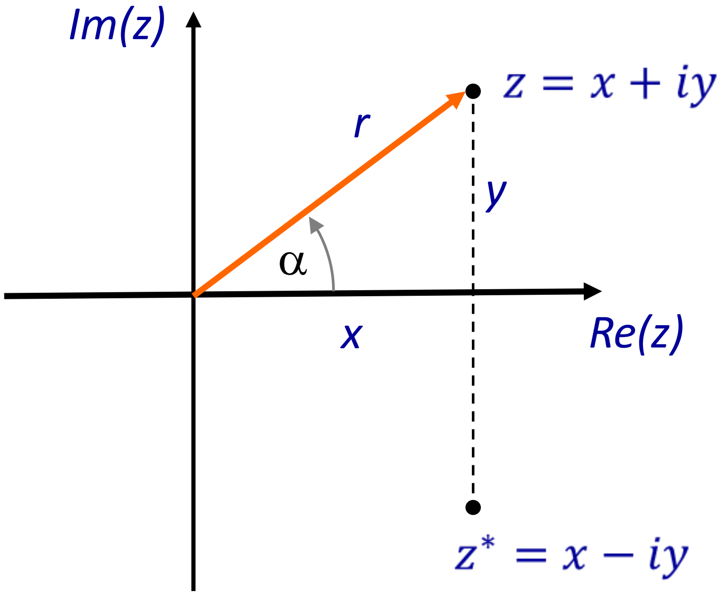
$z = x + iy$ $\qquad i^2 = -1 \qquad i = \sqrt{-1}$

$x = \mathrm{Re}(z)$

$y = \mathrm{Im}(z)$

$z = re^{i\alpha} \qquad r = \sqrt{x^2 + y^2}$

$$\tan\alpha = \frac{y}{x} = \frac{\mathrm{Re}(z)}{\mathrm{Im}(z)}$$

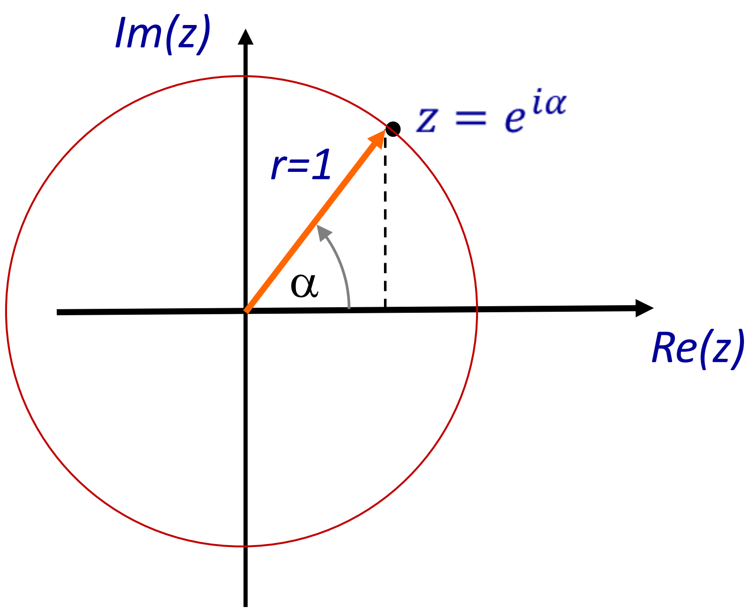


Einheitskreis

$$z = e^{i\alpha} = \cos\alpha + i\sin\alpha$$

$$\cos\alpha = \frac{1}{2}\left(e^{i\alpha} + e^{-i\alpha}\right)$$

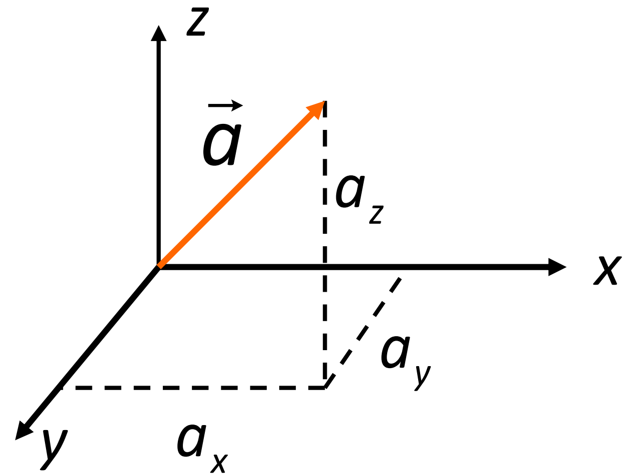
$$\sin\alpha = \frac{1}{2i}\left(e^{i\alpha} - e^{-i\alpha}\right)$$

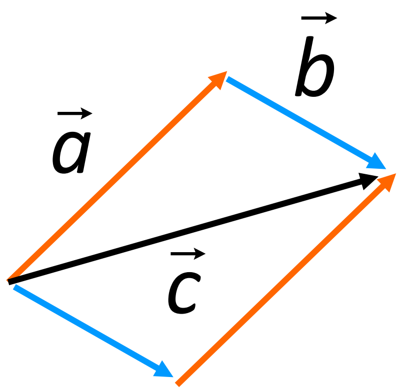


Vektoren

Skalare Größen: Betrag mit Einheit
Vektorielle Größen: Richtung und Betrag mit Einheit

$$\vec{a} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$$





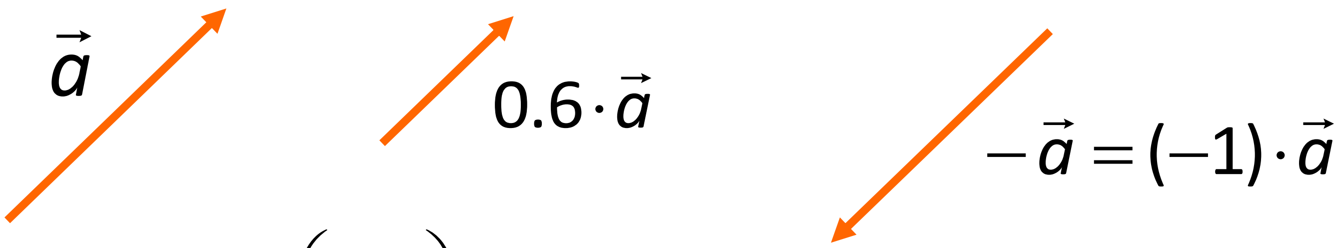
$$\vec{c} = \vec{a} + \vec{b}$$

$$\vec{c} = \vec{b} + \vec{a}$$

$$\vec{c} = \begin{pmatrix} a_x + b_x \\ a_y + b_y \\ a_z + b_z \end{pmatrix}$$

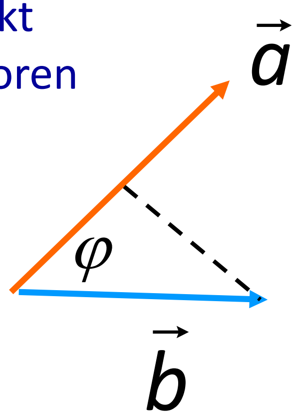
Vektoren

Multiplikation mit Skalar:



$$s \cdot \vec{a} = \begin{pmatrix} sa_x \\ sa_y \\ sa_z \end{pmatrix}$$

Skalarprodukt zweier Vektoren



$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

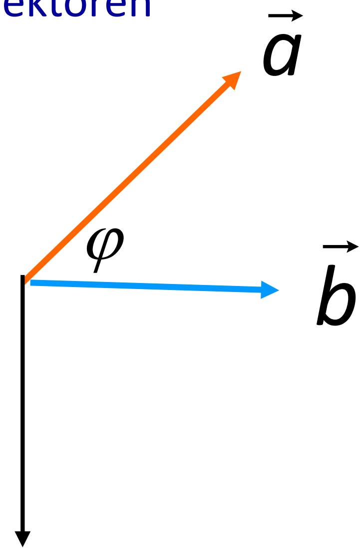
$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\varphi$$

$$\boxed{\vec{a} \cdot \vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}}$$

$$|\vec{a}| = \sqrt{\vec{a} \cdot \vec{a}}$$ Pythagoras

Vektoren

Kreuzprodukt
zweier Vektoren



$$\vec{c} \perp \vec{a} \text{ und } \vec{c} \perp \vec{b}$$

$$\vec{a} \times \vec{b} = \begin{pmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{pmatrix}$$

$$\vec{c} = \vec{a} \times \vec{b}$$

$$\vec{a} \times \vec{a} = 0$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$\left|\vec{c}\right| = \left|\vec{a}\right| \left|\vec{b}\right| \sin\varphi$$

wenn a und b
senkrecht zueinander

$$\left|\vec{c}\right| = \left|\vec{a}\right| \left|\vec{b}\right|$$

Gradient, Divergenz, Rotation

$$\vec{\nabla} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}$$

Nabla-Operator

Skalar

$$\vec{\nabla}\Phi = \mathrm{grad}\,\Phi = \begin{pmatrix} \frac{\partial \Phi}{\partial x} \\ \frac{\partial \Phi}{\partial y} \\ \frac{\partial \Phi}{\partial z} \end{pmatrix}$$

Gradient

Vektor

$$\vec{\nabla}\,\vec{u} = \mathrm{div}\,\vec{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}$$

Divergenz

Vektor

$$\vec{\nabla} \times \vec{u} = \mathrm{rot}\,\vec{u} = \begin{pmatrix} \frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z} \\ \frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x} \\ \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} \end{pmatrix}$$

Rotation

Skalar

Laplace

$$\vec{\nabla}^2\Phi = \Delta\,\Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2}$$