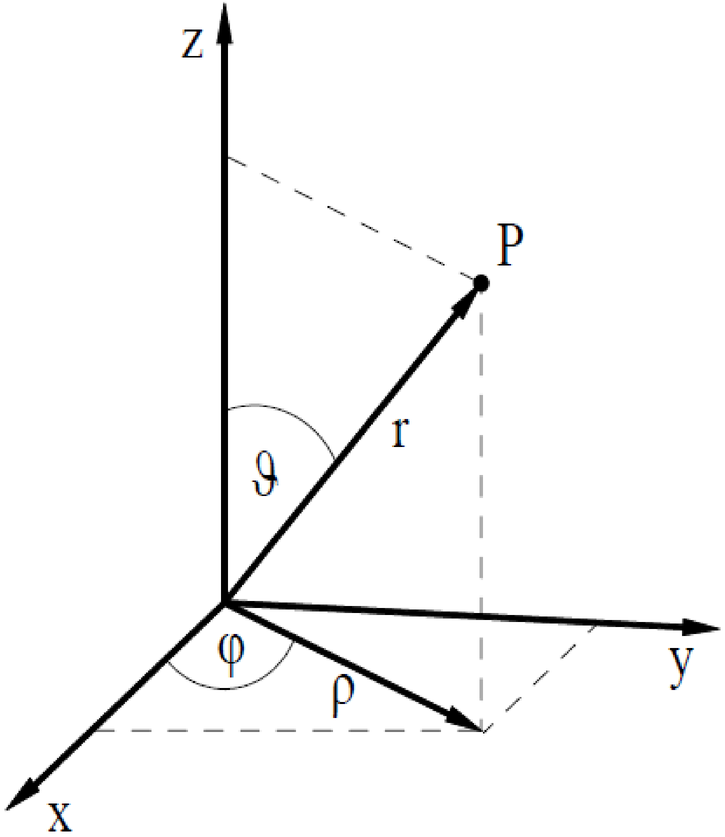
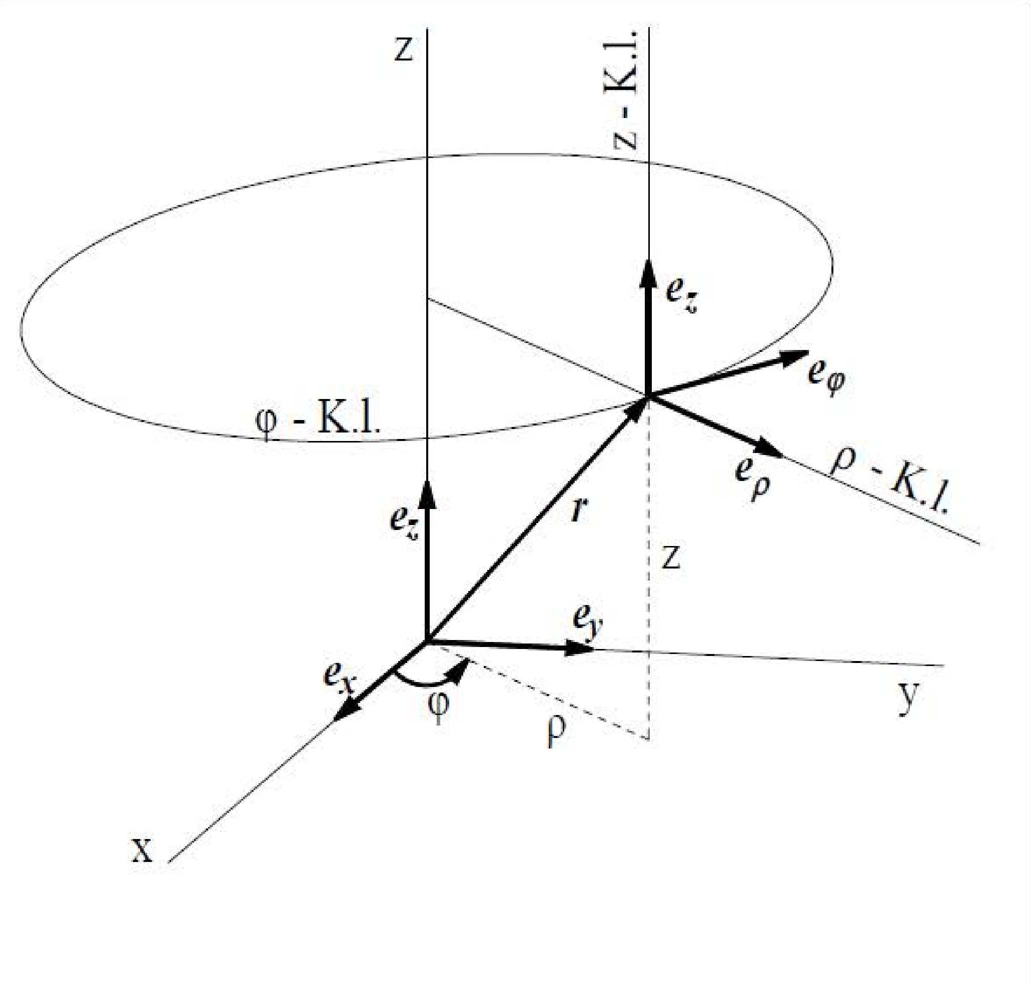
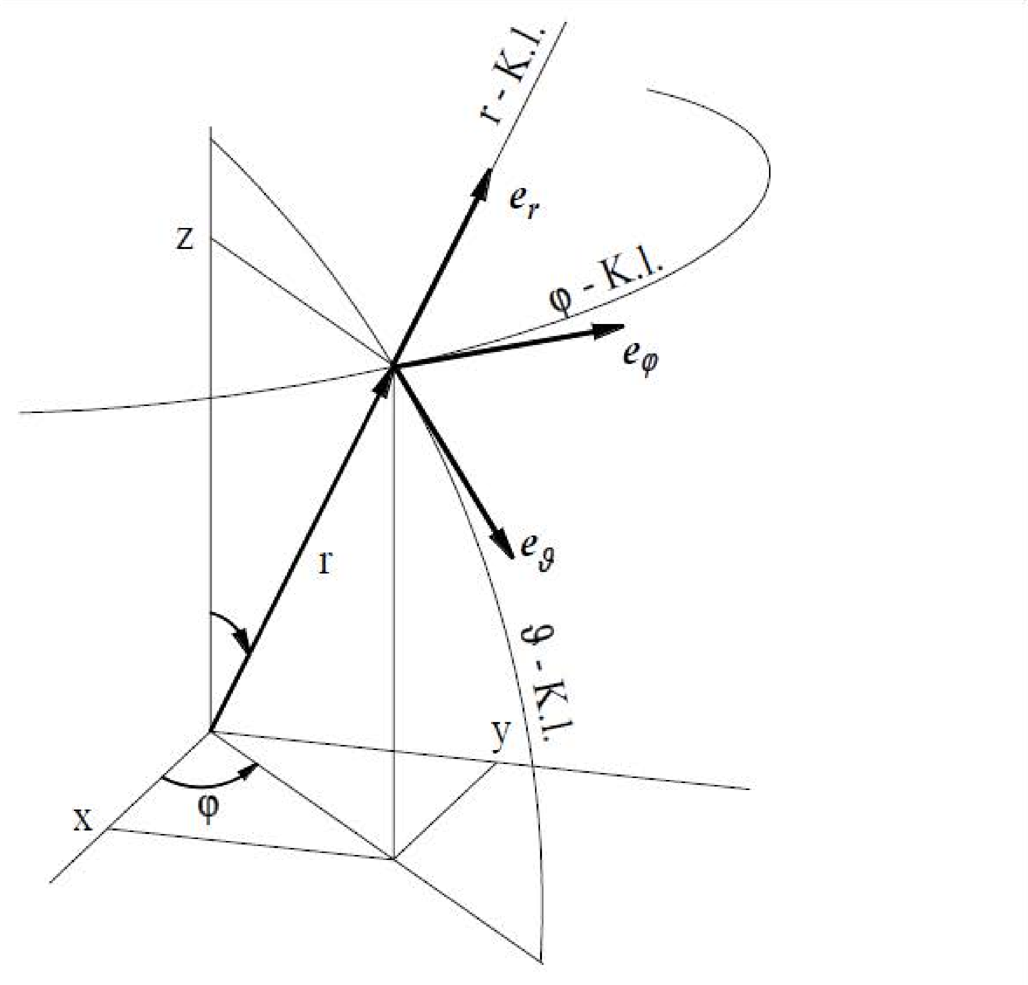


Kartesische Koordinaten	Zylinderkoordinaten	Kugelkoordinaten
x	$\rho \cos\varphi$	$r \sin\vartheta \cos\varphi$
y	$\rho \sin\varphi$	$r \sin\vartheta \sin\varphi$
z	z	$r \cos\vartheta$
$\sqrt{x^2 + y^2}$	ρ	$r \sin\vartheta$
$\arctan \frac{y}{x}$	φ	φ
z	z	$r \cos\vartheta$
$\sqrt{x^2 + y^2 + z^2}$	$\sqrt{\rho^2 + z^2}$	r
$\arctan \frac{\sqrt{x^2 + y^2}}{z}$	$\arctan \frac{\rho}{z}$	ϑ
$\arctan \frac{y}{x}$	φ	φ

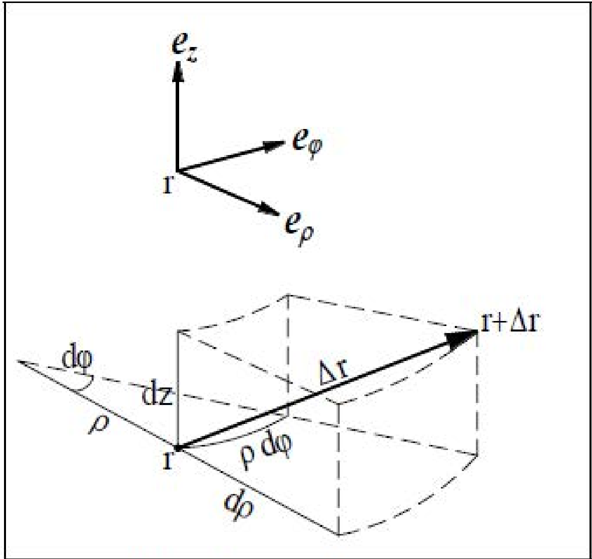


Zylinderkoordinaten : ρ, φ, z

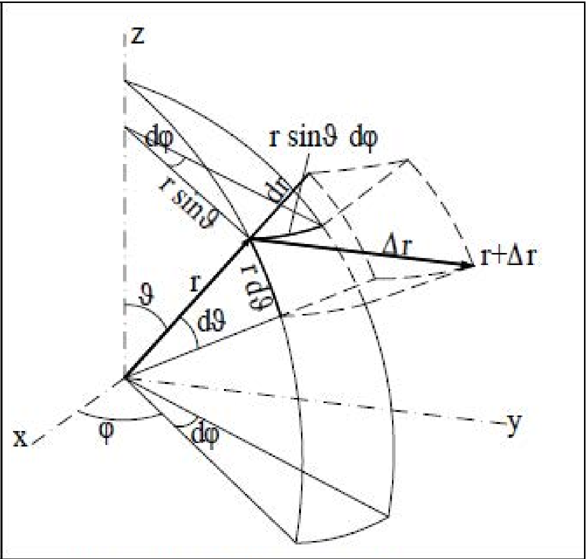


Kugelkoordinaten : r, ϑ, φ

K.l. : Koordinatenlinien mit den zugehörigen Basisvektoren; die Koordinatenlinien zeigen in Richtung wachsender Koordinaten



$\Delta r \approx dr = d\rho\ e_\rho + \rho\ d\varphi\ e_\varphi + dz\ e_z$



$\Delta r \approx dr = dr\ e_r + r\ d\vartheta\ e_\vartheta + r\ \sin\vartheta\ d\varphi\ e_\varphi$

Der Zusammenhang zwischen den Vektorkomponenten in verschiedenen Koordinatensystemen

Kartesische Koordinaten	Zylinderkoordinaten	Kugelkoordinaten
A_x A_y A_z	$A_\rho \cos\varphi - A_\varphi \sin\varphi$ $A_\rho \sin\varphi + A_\varphi \cos\varphi$ A_z	$A_r \sin\vartheta \cos\varphi + A_\vartheta \cos\vartheta \cos\varphi - A_\varphi \sin\varphi$ $A_r \sin\vartheta \sin\varphi + A_\vartheta \cos\vartheta \sin\varphi + A_\varphi \cos\varphi$ $A_r \cos\vartheta - A_\vartheta \sin\vartheta$
$A_x \cos\varphi + A_y \sin\varphi$ $-A_x \sin\varphi + A_y \cos\varphi$ A_z	A_ρ A_φ A_z	$A_r \sin\vartheta + A_\vartheta \cos\vartheta$ A_φ $A_r \cos\vartheta - A_\vartheta \sin\vartheta$
$A_x \sin\vartheta \cos\varphi + A_y \sin\vartheta \sin\varphi + A_z \cos\vartheta$ $A_x \cos\vartheta \cos\varphi + A_y \cos\vartheta \sin\varphi - A_z \sin\vartheta$ $-A_x \sin\varphi + A_y \cos\varphi$	$A_\rho \sin\vartheta + A_z \cos\vartheta$ $A_\rho \cos\vartheta - A_z \sin\vartheta$ A_φ	A_r A_ϑ A_φ

Setzt man an Stelle der Vektorkomponenten die Einheitsvektoren ein, so erhält man den Zusammenhang zwischen den Einheitsvektoren in den verschiedenen Koordinatensystemen.

	Kartesische Koordinaten	Zylinderkoordinaten	Kugelkoordinaten
grad Φ	$e_x \frac{\partial \Phi}{\partial x} + e_y \frac{\partial \Phi}{\partial y} + e_z \frac{\partial \Phi}{\partial z}$	$e_\rho \frac{\partial \Phi}{\partial \rho} + e_\varphi \frac{1}{\rho} \frac{\partial \Phi}{\partial \varphi} + e_z \frac{\partial \Phi}{\partial z}$	$e_r \frac{\partial \Phi}{\partial r} + e_\vartheta \frac{1}{r} \frac{\partial \Phi}{\partial \vartheta} + e_\varphi \frac{1}{r \sin\vartheta} \frac{\partial \Phi}{\partial \varphi}$
div A	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho A_\rho\right) + \frac{1}{\rho} \frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r \sin\vartheta} \frac{\partial}{\partial \vartheta}\left(A_\vartheta \sin\vartheta\right) + \frac{1}{r \sin\vartheta} \frac{\partial A_\varphi}{\partial \varphi}$
rot A	$\begin{aligned} & e_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \\ & + e_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \\ & + e_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) \end{aligned}$	$\begin{aligned} & e_\rho \left(\frac{1}{\rho} \frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right) + \\ & + e_\varphi \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right) + \\ & + e_z \left(\frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho A_\varphi\right) - \frac{1}{\rho} \frac{\partial A_\rho}{\partial \varphi}\right) \end{aligned}$	$\begin{aligned} & e_r \frac{1}{r \sin\vartheta} \left(\frac{\partial}{\partial \vartheta}\left(A_\varphi \sin\vartheta\right) - \frac{\partial A_\vartheta}{\partial \varphi}\right) + \\ & + e_\vartheta \frac{1}{r} \left(\frac{1}{\sin\vartheta} \frac{\partial A_r}{\partial \varphi} - \frac{\partial}{\partial r}\left(r A_\varphi\right)\right) + \\ & + e_\varphi \frac{1}{r} \left(\frac{\partial}{\partial r}\left(r A_\vartheta\right) - \frac{\partial A_r}{\partial \vartheta}\right) \end{aligned}$
$\Delta\Phi$	$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2}$	$\frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho \frac{\partial \Phi}{\partial \rho}\right) + \frac{1}{\rho^2} \frac{\partial^2 \Phi}{\partial \varphi^2} + \frac{\partial^2 \Phi}{\partial z^2}$	$\frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial \Phi}{\partial r}\right) + \frac{1}{r^2 \sin\vartheta} \frac{\partial}{\partial \vartheta}\left(\sin\vartheta \frac{\partial \Phi}{\partial \vartheta}\right) + \frac{1}{r^2 \sin^2\vartheta} \frac{\partial^2 \Phi}{\partial \varphi^2}$
ds	$e_x\, dx + e_y\, dy + e_z\, dz$	$e_\rho\, d\rho + e_\varphi\, \rho\, d\varphi + e_z\, dz$	$e_r\, dr + e_\vartheta\, r\, d\vartheta + e_\varphi\, r \sin\vartheta\, d\varphi$

$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}\,(\mathbf{A}\,\mathbf{C}) - \mathbf{C}\,(\mathbf{A}\,\mathbf{B})$	
$\text{grad}\,(\Phi + \Psi) = \text{grad}\,\Phi + \text{grad}\,\Psi$ $\text{grad}\,(c\Phi) = c\,\text{grad}\,\Phi \qquad c = \text{const.}$ $\text{grad}\,(\Phi\Psi) = \Phi\,\text{grad}\,\Psi + \Psi\,\text{grad}\,\Phi$ $\text{grad}\,(\mathbf{A}\mathbf{B}) = (\mathbf{A}\,\text{grad})\,\mathbf{B} + (\mathbf{B}\,\text{grad})\,\mathbf{A} + \mathbf{A} \times \text{rot}\,\mathbf{B} + \mathbf{B} \times \text{rot}\,\mathbf{A}$ $\text{grad}\,r = \frac{\mathbf{r}}{r} = \mathbf{e}_r \quad \text{mit} \quad r = \sqrt{x^2+y^2+z^2}$ $\text{grad}\,[\Phi(r)] = \Phi'(r)\,\mathbf{e}_r$ $\text{grad}\,\frac{1}{r} = -\frac{\mathbf{r}}{r^3} = -\frac{1}{r^2}\mathbf{e}_r$ $\text{grad}\,[\,\ln(r)\,] = \frac{\mathbf{r}}{r^2} = \frac{1}{r}\mathbf{e}_r$ $\text{div}\,(\mathbf{A} + \mathbf{B}) = \text{div}\,\mathbf{A} + \text{div}\,\mathbf{B}$ $\text{div}\,(c\mathbf{A}) = c\,\text{div}\,\mathbf{A} \qquad c = \text{const.}$ $\text{div}\,(\Phi\mathbf{A}) = \Phi\,\text{div}\,\mathbf{A} + \mathbf{A}\,\text{grad}\,\Phi$ $\text{div}\,(\mathbf{A} \times \mathbf{B}) = \mathbf{B}\,\text{rot}\,\mathbf{A} - \mathbf{A}\,\text{rot}\,\mathbf{B}$ $\text{div}\,\mathbf{e}_r = \frac{2}{r}$ $\text{div}\,[\,\Phi(r)\,\mathbf{r}\,] = 3\,\Phi(r) + r\,\Phi'(r)$ $\text{div}\,\text{rot}\,\mathbf{A} = 0$	$\text{rot}\,(\mathbf{A} + \mathbf{B}) = \text{rot}\,\mathbf{A} + \text{rot}\,\mathbf{B} \qquad \text{rot}\,(c\mathbf{A}) = c\,\text{rot}\,\mathbf{A}$ $\text{rot}\,(\Phi\mathbf{A}) = \Phi\,\text{rot}\,\mathbf{A} + \text{grad}\,\Phi \times \mathbf{A}$ $\text{rot}\,(\mathbf{A} \times \mathbf{B}) = \mathbf{A}\,\text{div}\,\mathbf{B} - \mathbf{B}\,\text{div}\,\mathbf{A} + (\mathbf{B}\,\text{grad})\,\mathbf{A} - (\mathbf{A}\,\text{grad})\,\mathbf{B}$ $\text{rot}\,\text{rot}\,\mathbf{A} = \text{grad}\,\text{div}\,\mathbf{A} - \Delta\mathbf{A}$ $\text{rot}\,[\Phi(r)\,\mathbf{r}\,] = 0$ $\text{rot}\,(\text{grad}\,\Phi) = 0$ $\text{rot}\,\mathbf{e}_r = 0$ $(\mathbf{A}\,\text{grad})\,\mathbf{B} = (\mathbf{A}\,\text{grad}\,B_x)\,\mathbf{e}_x + (\mathbf{A}\,\text{grad}\,B_y)\,\mathbf{e}_y + (\mathbf{A}\,\text{grad}\,B_z)\,\mathbf{e}_z$ $\Delta\mathbf{A} = \mathbf{e}_x\,\Delta A_x + \mathbf{e}_y\,\Delta A_y + \mathbf{e}_z\,\Delta A_z = \mathbf{e}_x\left(\frac{\partial^2 A_x}{\partial x^2}+\frac{\partial^2 A_x}{\partial y^2}+\frac{\partial^2 A_x}{\partial z^2}\right) +$ $+\,\mathbf{e}_y\left(\frac{\partial^2 A_y}{\partial x^2}+\frac{\partial^2 A_y}{\partial y^2}+\frac{\partial^2 A_y}{\partial z^2}\right) + \mathbf{e}_z\left(\frac{\partial^2 A_z}{\partial x^2}+\frac{\partial^2 A_z}{\partial y^2}+\frac{\partial^2 A_z}{\partial z^2}\right)$ <u>STOKESscher Integralsatz</u> $\oint \mathbf{A}\,d\mathbf{s} = \iint \text{rot}\,\mathbf{A}\,d\mathbf{f}$ <u>GAUSSscher Integralsatz</u> $\oint\!\!\oint \mathbf{A}\,d\mathbf{f} = \iiint \text{div}\,\mathbf{A}\,dV$