Interacting multi-component exciton gases in a Hertzian potential trap

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Introduction

Motivation:
Current experiments by H. Stolz: measurements of the decay luminescence of trapped excitons in cuprous oxide require

- development of the theoretical framework
- interpretation of experimental results
- predictions for future experiments

System:
Mixture of paraexcitons and two species of orthoexcitons $(+/-)$ in cuprous oxide, trapped in a stress induced Hertzian potential

Object of interest:
Spatially resolved decay luminescence spectrum of each species and signatures of Bose-Einstein condensation therein

Model:

- multicomponent gas of structureless bosons [1]
- intra- and inter-species interaction via contact potential (s-wave scattering)
- external potential fitted to experimental data

Theory I: Hartree-Fock-Bogoliubov-Popov equations

Starting from the Hamiltonian for a K-component system

$$\hat{H} = \sum_i \int d\mathbf{r}\, \hat{\psi}_i^\dagger(\mathbf{r},t)\left(-\frac{\hbar^2\nabla^2}{2m_i} + V_i(\mathbf{r}) - \mu_i\right)\hat{\psi}_i(\mathbf{r},t) + \frac{1}{2}\sum_{i,j=1}^{K}\int d\mathbf{r}\, h_{ij}\hat{\psi}_i^\dagger(\mathbf{r},t)\hat{\psi}_j^\dagger(\mathbf{r},t)\hat{\psi}_j(\mathbf{r},t)\hat{\psi}_i(\mathbf{r},t)\,,$$

we decompose the Bose field operator $\hat{\psi}_i(\mathbf{r},t) = \Phi_i(\mathbf{r}) + \tilde{\psi}_i(\mathbf{r},t)$ and derive the Gross-Pitaevskii equation for the condensate wave function of species i:

$$0 = \left(-\frac{\hbar^2\nabla^2}{2m_i} + V_i - \mu_i + h_{ii}\left(|\Phi_i|^2 + 2\tilde{n}_{ii}\right) + \sum_{j\neq i} h_{ij}n_{jj}\right)\Phi_i + h_{ii}\tilde{m}_{ii}\Phi_i^* + \sum_{j\neq i} h_{ij}\left(\tilde{n}_{ji}\Phi_j + \tilde{m}_{ji}\Phi_j^*\right).$$

The equation of motion for thermal excitons follows as

$$i\hbar\frac{\partial\tilde{\psi}_i}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m_i} + V_i - \mu_i + 2h_{ii}n_{ii} + \sum_{j\neq i} h_{ij}n_{jj}\right)\tilde{\psi}_i + h_{ii}m_{ii}\tilde{\psi}_i^\dagger + \sum_{j\neq i} h_{ij}\left(n_{ij}\tilde{\psi}_j + m_{ij}\tilde{\psi}_j^\dagger\right),$$

with $n_{ij} \equiv \Phi_j^*\Phi_i + \tilde{n}_{ij}$, $m_{ij} \equiv \Phi_j\Phi_i + \tilde{m}_{ij}$, and the normal and anomal averages $\tilde{n}_{ij} = \langle\tilde{\psi}_i^\dagger\tilde{\psi}_j\rangle$ and $\tilde{m}_{ij} = \langle\tilde{\psi}_i\tilde{\psi}_j\rangle$, respectively. h_{ij} denotes the interaction strength given by the s-wave scattering length [2].

For simplicity, we neglect all non-diagonal averages, i.e. $\tilde{m}_{ij} = \tilde{n}_{ij} = m_{ij} = n_{ij} = 0 \quad \forall i \neq j$, and arrive at effective 1-species equations.

In the local-density approximation, a Bogoliubov transformation with the amplitudes u_i, v_i gives the quasiparticle energies in the HFB-Popov limit ($\tilde{m}_{ii} \to 0$) as

$$E_i = \sqrt{\mathfrak{L}_i^2 - (h_{ii}n_i^c)^2}\,,$$

with the condensate density $n_i^c \equiv |\Phi_i|^2$ and

$$\mathfrak{L}_i = \frac{p^2}{2m_i} + V_i - \mu_i + 2h_{ii}(n_i^c + n_i^T) + \sum_{j\neq i} h_{ij}(n_j^c + n_j^T)\,.$$

The non-condensate density $n_i^T \equiv \tilde{n}_{ii}$ reads

$$n_i^T = \int\frac{d^3\mathbf{p}}{(2\pi\hbar)^3}\left[\frac{\mathfrak{L}_i}{E_i}\left(n_B(E_i) + \frac{1}{2}\right) - \frac{1}{2}\right]\Theta\left(E_i^2\right),$$

while a Thomas-Fermi approximation to the GPE yields the condensate density as

$$n_i^c = \frac{1}{h_{ii}}\left[\mu_i - V_i - 2h_{ii}n_i^T - \sum_{j\neq i} h_{ij}(n_j^c + n_j^T)\right] \times \Theta\left(\mu_i - V_i - 2h_{ii}n_i^T - \sum_{j\neq i} h_{ij}(n_j^c + n_j^T)\right).$$

$\Rightarrow$ system of 2K coupled equations

Theory II: Decay luminescence spectrum

According to [3], the local spectral intensity is expressed as

$$I(\mathbf{r},\omega) = 2\pi|S_{\mathbf{k}=0}|^2 n^c(\mathbf{r})\,\delta(\omega'-\mu) + \sum_{\mathbf{k}\neq 0}|S_{\mathbf{k}}|^2 n_B(\omega'-\mu)A(\mathbf{r},\mathbf{k},\omega'-\mu)\,,$$

with the excitonic spectral function in LDA

$$A(\mathbf{r},\mathbf{k},\omega) = 2\pi\left[u^2(\mathbf{k},\mathbf{r})\delta(\hbar\omega - E(\mathbf{k},\mathbf{r})) - v^2(\mathbf{k},\mathbf{r})\delta(\hbar\omega + E(\mathbf{k},\mathbf{r}))\right].$$

Considering the spectral resolution Δ and finite slit width $2\Delta x$ of an experimental setup, yields the phonon-assisted spectrum for thermal orthoexcitons:

$$I_{PA}(z,\omega') \propto \int_{-\Delta x}^{\Delta x} dx \int_{-\infty}^{\infty} dy \int d\mathbf{k}\, u^2(\mathbf{k},\mathbf{r})n_B(E(\mathbf{k},\mathbf{r}))\exp\left[-\varepsilon_-^4(\omega',\mathbf{k},\mathbf{r})\right] - \int_{-\Delta x}^{\Delta x} dx \int_{-\infty}^{\infty} dy \int d\mathbf{k}\, v^2(\mathbf{k},\mathbf{r})n_B(-E(\mathbf{k},\mathbf{r}))\exp\left[-\varepsilon_+^4(\omega',\mathbf{k},\mathbf{r})\right],$$

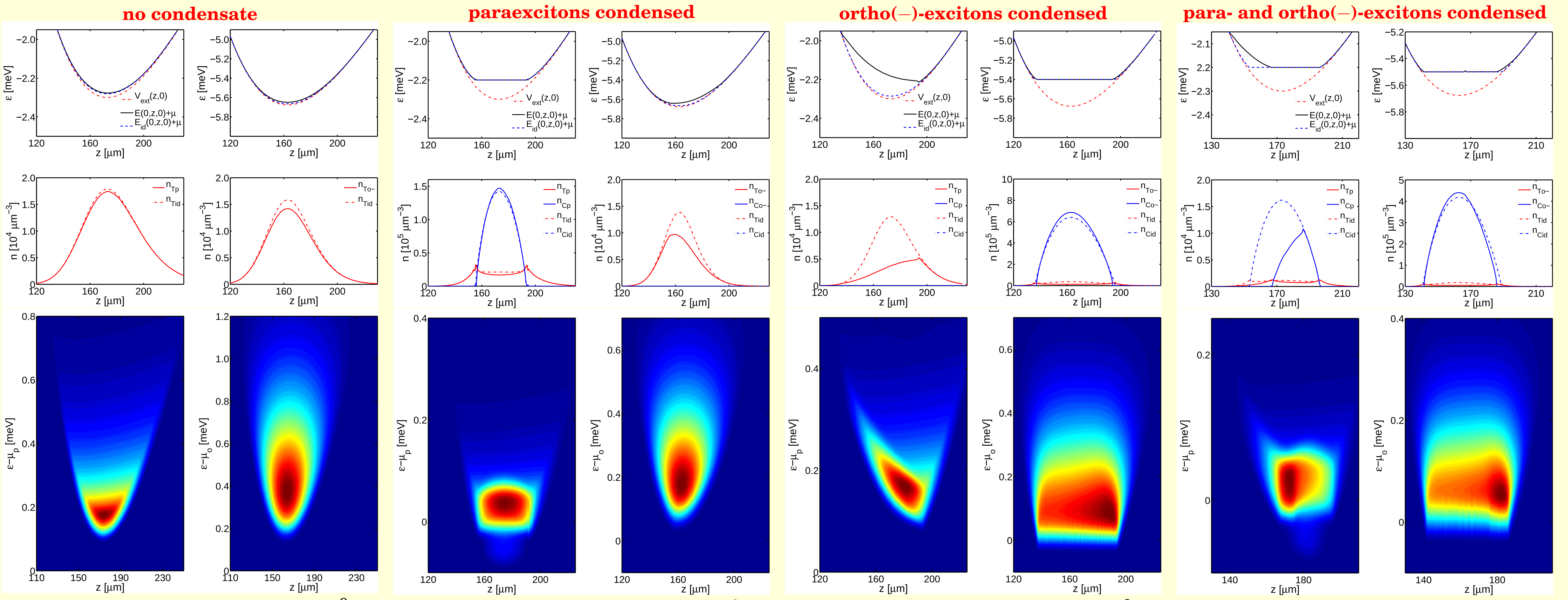
with $\varepsilon_\pm(\omega',\mathbf{k},\mathbf{r}) \equiv (\hbar\omega' - \mu \pm E(\mathbf{k},\mathbf{r}))/\Delta$ and $\omega' = \omega + \omega_{ph} - \omega_{gX}$. The direct decay spectrum for paraexcitons ($\omega_{ph} = 0$) results from setting $S_{\mathbf{k}} \propto \delta(\mathbf{k} - \mathbf{k}_0)$ with the photon wave vector $\mathbf{k}_0$ [4]

$$I_{ZP}(z,\omega') \propto \int_{-\Delta x}^{\Delta x} dx \int_{-\infty}^{\infty} dy\, u^2(\mathbf{k}_0,\mathbf{r})n_B(E(k_0,\mathbf{r}))\exp\left[-\varepsilon_-^4(\omega',\mathbf{k}_0,\mathbf{r})\right] - \int_{-\Delta x}^{\Delta x} dx \int_{-\infty}^{\infty} dy\, v^2(\mathbf{k}_0,\mathbf{r})n_B(-E(k_0,\mathbf{r}))\exp\left[-\varepsilon_+^4(\omega',\mathbf{k}_0,\mathbf{r})\right].$$

Numerical results

Upper row: trap potentials and renormalized potentials; **middle row:** exciton densities; **lower row:** luminescence spectra of thermal para- (left) and orthoexcitons (right)

no condensate | paraexcitons condensed | ortho(−)-excitons condensed | para- and ortho(−)-excitons condensed



Parameter: T=2.0 K; $N_p = 1.64\times10^9$; $N_{o-} = N_{o+} = 6.90\times10^8$

Parameter: T=1.2 K; $N_p = 2.75\times10^9$; $N_{o-} = N_{o+} = 2.28\times10^8$

Parameter: T=1.2 K; $N_p = 3.42\times10^8$; $N_{o-} = 1.93\times10^{10}$; $N_{o+} = 1.32\times10^9$

Parameter: T=0.7 K; $N_p = 1.10\times10^9$; $N_{o-} = 6.06\times10^9$; $N_{o+} = 2.70\times10^8$

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